Adaptive Temporal Decorrelation of State Estimates

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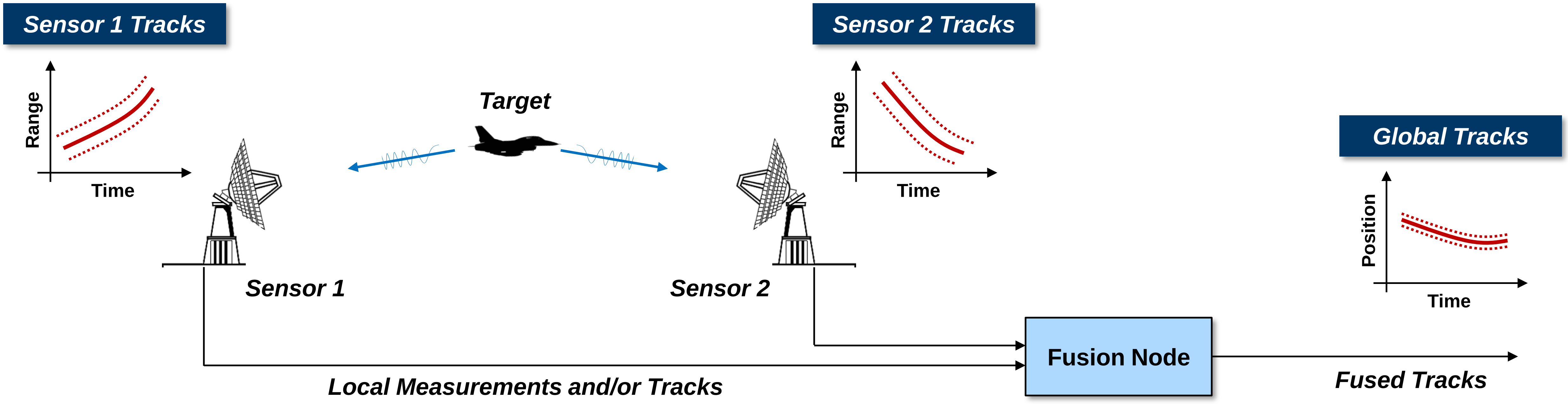
Distributed Tracking Architectures



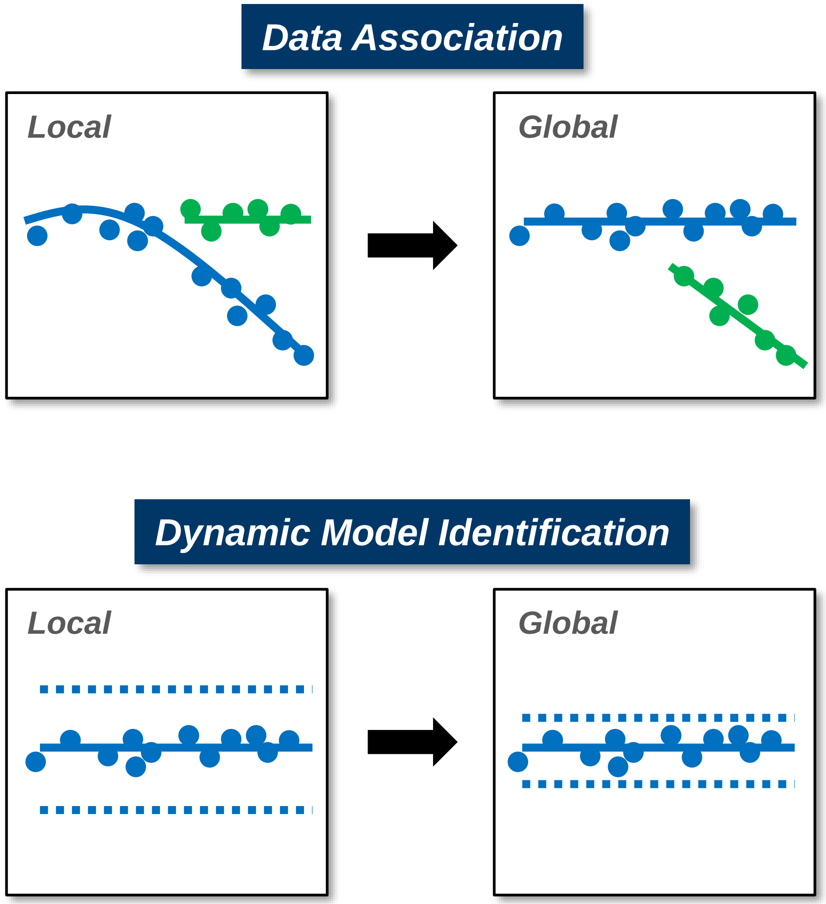
- **Distributed tracking architectures fuse local tracking information from multiple sensors at a fusion node to create global tracks**
  - Combining tracking and/or measurement information across sensors can increase accuracy and data association performance through varied geometry and modalities
- **Depending on constraints of sensor network, sensor information may by limited in many different ways, leading to a rich literature on effective distributing tracking[1]**

[1] C.-Y. Chong, K.-C. Chang, and S. Mori, "A review of forty years of distributed estimation," in *Proceedings of International Conference on Information Fusion*, 2018, pp. 1–8

Reassociation and Refiltering at Fusion Node

- **Noteworthy special case of distributed tracking is the need to *refilter* local track information based on the information history accumulated at the fusion node**
  - Disambiguated data association
  - Identification of motion model regimes
  - Correction of local sensor tracking errors

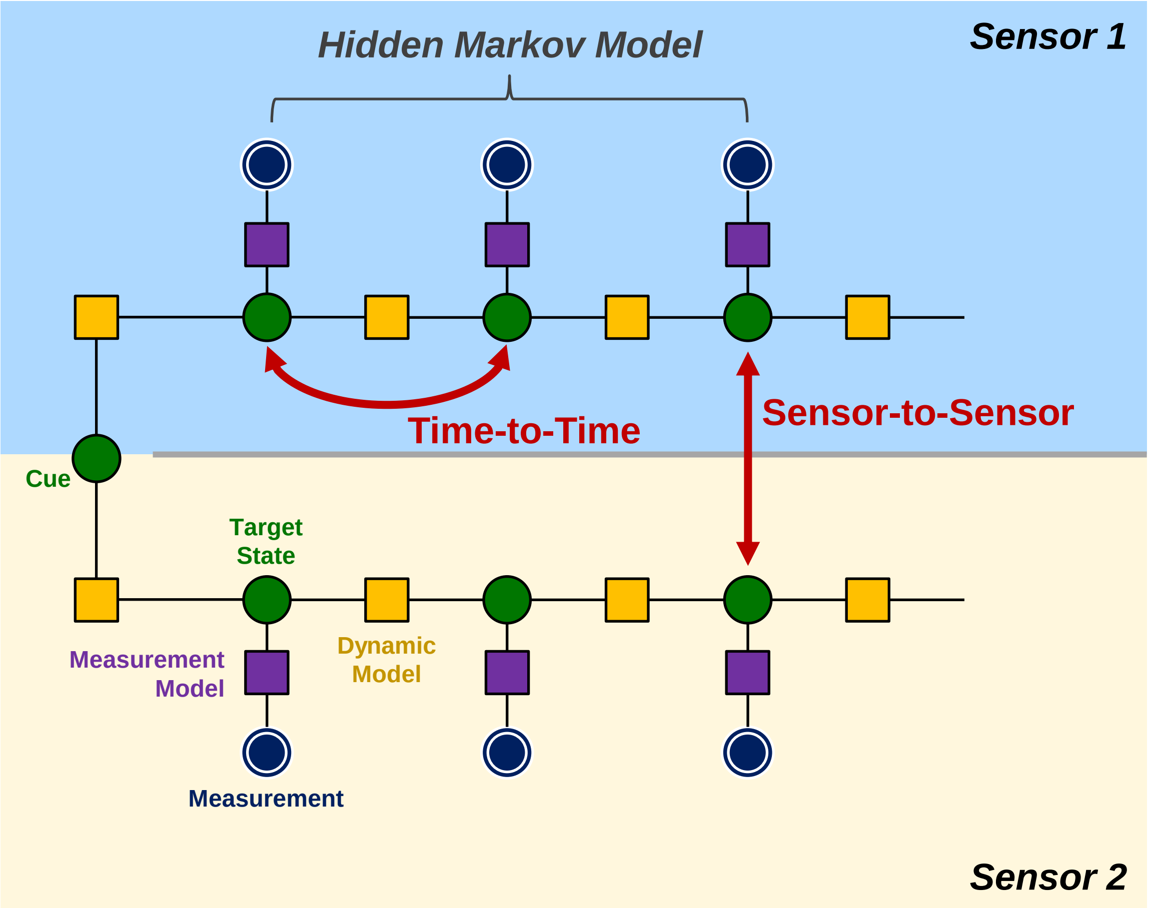
- **Fusion node can act as a “macro-tracker” and refine local state estimates[1,2]**



[1]C.-Y. Chong, “Graph approaches for data association,” in *Proceedings of International Conference on Information Fusion*, 2012, pp. 1–8
[2]G. Castañón and L. Finn, “Multi-target tracklet stitching through network flows,” in *Proceedings of IEEE Aerospace Conference*, 2011

Refiltering Using Track State Information

Correlations in Sensor Tracks

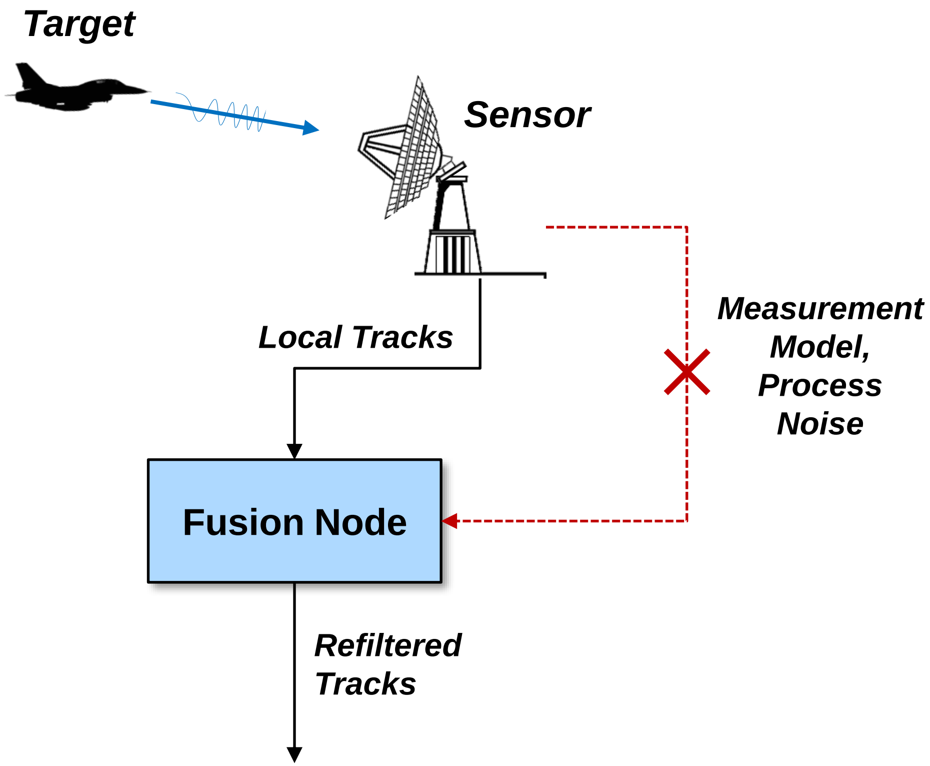


- **Practical sensor networks often have strict communications limits and are confined to sending only track state information to a fusion node**
- **Track states from sensors can be correlated in multiple different ways[1]:**
  - **Time-to-time (temporal):** A track state from one time step to another is related due to recursive filtering
  - **Sensor-to-sensor:** A track state at a different sensor can have shared information, e.g., commonly-observed maneuver, prior initialization information
- **Before refiltering, correlation must be accounted for; focus of this talk is *addressing temporal correlation to enable refiltering at a fusion node***
  - If unaccounted for, correlation will degrade estimates and produce optimistic/pessimistic covariances

[1]C.-Y. Chong, K.-C. Chang, and S. Mori, "A review of forty years of distributed estimation," in *Proceedings of International Conference on Information Fusion*, 2018, pp. 1–8

Decorrelation of State Estimates

- **Decorrelation across time and/or sensor can be done via learning or modeling the correlation and removing it (i.e., pre-whitening); approached many ways, e.g.,**
  - Tracklet fusion[1]
  - Distributed Kalman filtering[2]
  - Adaptive linear estimation[3]
- **Focus on a model-based approach to decorrelation of state estimates with non-zero process noise from time-to-time for a sensor**
  - Employs a pseudomeasurement[4] formulation with conservative estimation of unknown measurement model and process noise parameters; zero process noise case treated previously[5]



[1]C.-Y. Chong, etal., "Architectures and algorithms for track association and fusion," *IEEE Aerospace and Electronics Systems Magazine*, vol. 15, no. 1, pp. 5–13, 2000
[2]W. Koch, "Exact update formulae for distributed Kalman filtering and retrodiction at arbitrary communication rates," *Proceedings of International Conference on Information Fusion*, Seattle, WA, 2009
[3]X. R. Li, Y. Zhu, and C. Han, "Unified optimal linear estimation fusion – Part I: Unified models and fusion results," *Proceedings of International Conference on Information Fusion*, Paris, France, 2000
[4]L. Y. Pao, "Measurement reconstruction approach for distributed multisensor fusion," Journal of Guidance, Control, and Dynamics, vol. 19, no. 4, pp. 842–847, 1996
[5]G. Frenkel, "Multisensor tracking of ballistic targets," in Signal andData Processing of Small Targets, O. E. Drummond, Ed., vol. 2561,International Society for Optics and Photonics. SPIE, 1995, pp. 337– 346

Outline

- Introduction
- Adaptive Temporal Correlation to Enable Refiltering
- Results
- Summary

Distributed Tracking System and Assumptions

- **Goal: Decorrelate state estimates from a single sensor to allow refiltering**
  - Fusion node only has access to state estimates and state estimate covariances
  - Dynamic model is *known*
  - Local measurement model is *unknown*
  - Process noise is *known* (for now)
  - Fusion node receives data at measurement rate (discussed later)
- **Approach:**
  - Reconstruct effective measurement information from track states[1] in a global coordinate system (similar to bookkeeping a global information gain)
  - Refilter effective measurement information

Target Dynamics:

$$\mathbf{v}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k-1})$$

$$\underbrace{\mathbf{x}_k}_{\text{Target State}} = \underbrace{f}_{\text{Dynamics}}(\mathbf{x}_{k-1}) + \underbrace{\mathbf{v}_{k-1}}_{\text{Process Noise}}$$

Sensor Measurements:

$$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$$

$$\underbrace{\mathbf{z}_k}_{\text{Measurement}} = \underbrace{h}_{\text{Measurement Mapping}}(\mathbf{x}_k) + \underbrace{\mathbf{w}_k}_{\text{Measurement Noise}}$$

Extended Kalman filter (EKF):

$$\hat{\mathbf{x}}_{k|k-1} = f\left(\hat{\mathbf{x}}_{k-1|k-1}\right)$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$$

$$\text{State Estimate} \longrightarrow \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{z}_k - h\left(\hat{\mathbf{x}}_{k|k-1}\right)\right)$$

$$\text{Estimate Covariance} \longrightarrow \mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T$$

$$\text{Kalman Gain} \longrightarrow \mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1}$$

$$\text{Innovation Covariance} \longrightarrow \mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

[1] L. Y. Pao, "Measurement reconstruction approach for distributed multisensor fusion," Journal of Guidance, Control, and Dynamics, vol. 19, no. 4, pp. 842–847, 1996

State Space Equivalent Measurements (SSEM)

- **Without knowledge of the sensor measurement model, goal is to construct effective measurement information *in the same coordinate system as the state space***
- **Assuming measurement matrix is identity, one can solve for the effective measurement vector and covariance using Lemma 1**
  - Measurement rank could also be estimated numerically if unknown
  - Some matrix conditioning may be necessary for reliable inversions
- **With a method to reconstruct measurements, now look at unknown process noise**

Lemma 1. *Given a dynamics function, $f(\cdot)$, process noise matrices, $\mathbf{Q}_{k-1}$, and the rank of the original measurements, M, the state space equivalent measurements from a series of state estimates and covariances can be derived by:*

1. *Construct propagated past state estimates, $\hat{\mathbf{x}}_{k|k-1}$, and covariances matrices, $\mathbf{P}_{k|k-1}$, by*

$$\hat{\mathbf{x}}_{k|k-1} = f\left(\hat{\mathbf{x}}_{k-1|k-1}\right),$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1},$$

2. *Obtain the inverse measurement covariance matrices, $\mathbf{R}_k^{-1}$, using*

$$\begin{bmatrix} \mathbf{R}_k^{-1} & \mathbf{0}_{M\times(N-M)} \\ \mathbf{0}_{(N-M)\times M} & \mathbf{0}_{(N-M)\times(N-M)} \end{bmatrix} = \mathbf{P}_{k|k}^{-1} - \mathbf{P}_{k|k-1}^{-1},$$

3. *Calculate the Kalman gain matrices, $\mathbf{K}_k$, with*

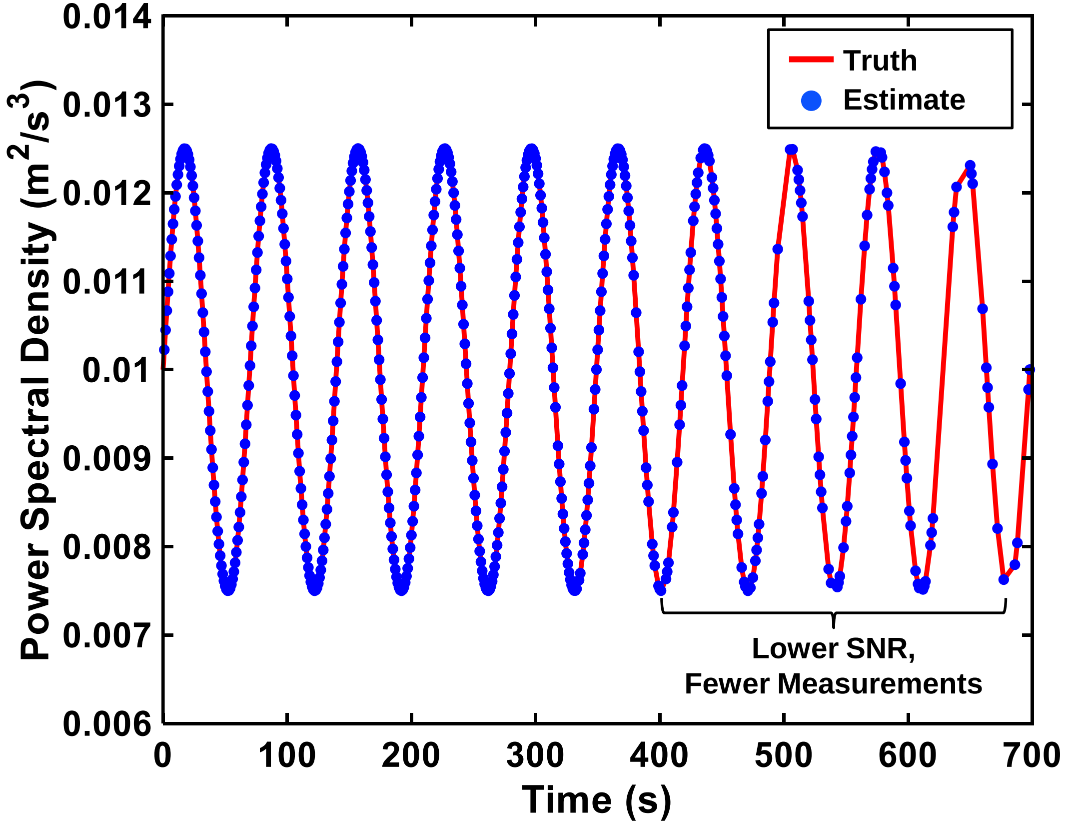
$$\left[\mathbf{K}_k \ \mathbf{0}_{N\times(N-M)}\right] = \mathbf{I}_N - \mathbf{P}_{k|k} \mathbf{P}_{k|k-1}^{-1},$$

4. *Create measurement vectors, $\mathbf{z}_k$, from*

$$\mathbf{z}_k = \mathbf{K}_k^{\dagger} \left(\hat{\mathbf{x}}_{k|k} - \mathbf{P}_{k|k} \mathbf{P}_{k|k-1}^{-1} \hat{\mathbf{x}}_{k|k-1}\right).$$

Conservative Estimation of Process Noise

Example: Spherically-Distributed White Process Noise of Unknown Power Spectral Density



- **Now, assume a process noise model is known to the fusion node up to a set of unknown parameters:**

$$\mathbf{Q}_{k-1}(\boldsymbol{\theta}_{k-1})$$

Process Noise Parameters

- **Process noise parameters can be conservatively estimated at the fusion node by achieving a minimum feasible information gain[1], i.e., choose θ such that**

$$\boxed{\mathbf{J}_k\left(\boldsymbol{\theta}'\right) \succeq \mathbf{J}_k\left(\hat{\boldsymbol{\theta}}\right) \succeq \mathbf{0}, \text{for all values of } \boldsymbol{\theta}' \neq \hat{\boldsymbol{\theta}}}$$

- **Total decorrelation process is to form SSEM and then estimate process noise using above**

[1]Information gain matrix is $\mathbf{J}_k = \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k$

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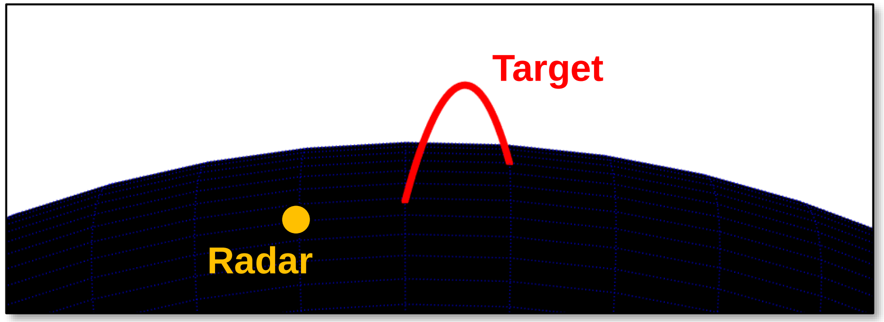
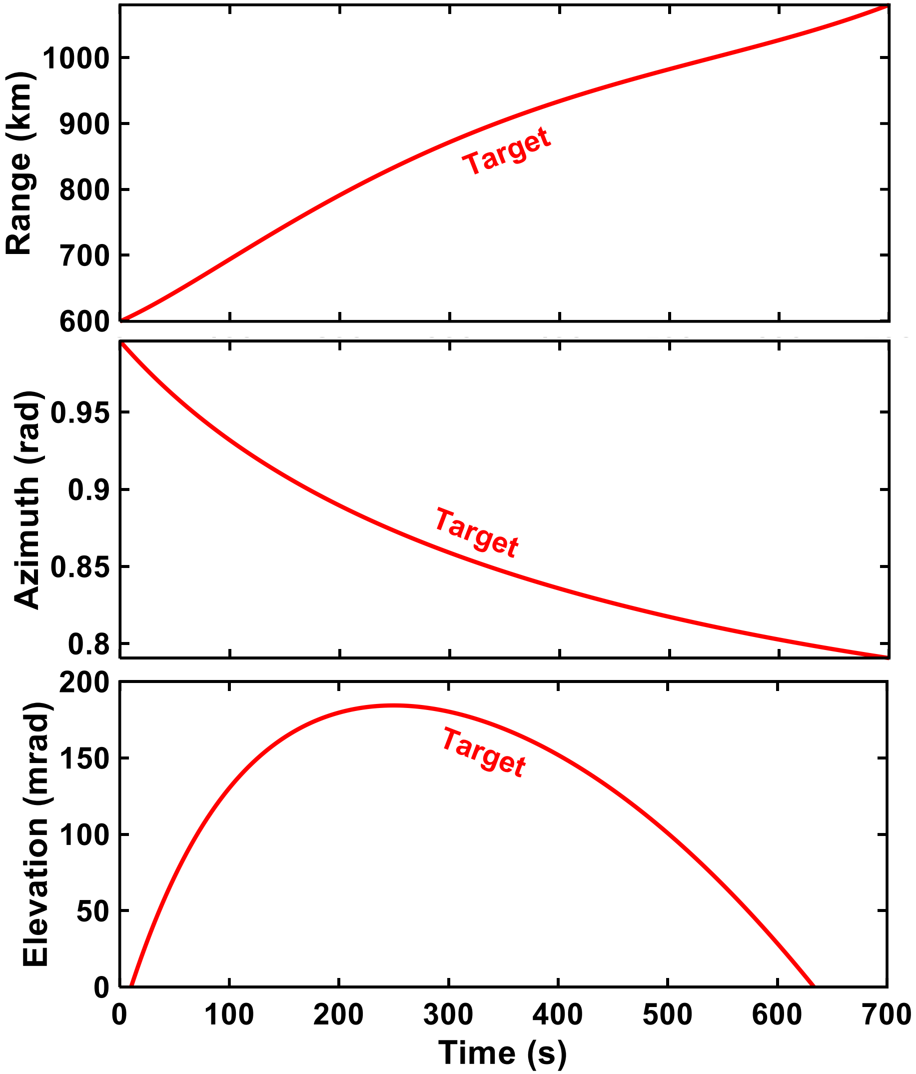
Ballistic Target Tracking Scenario

Example Scenario

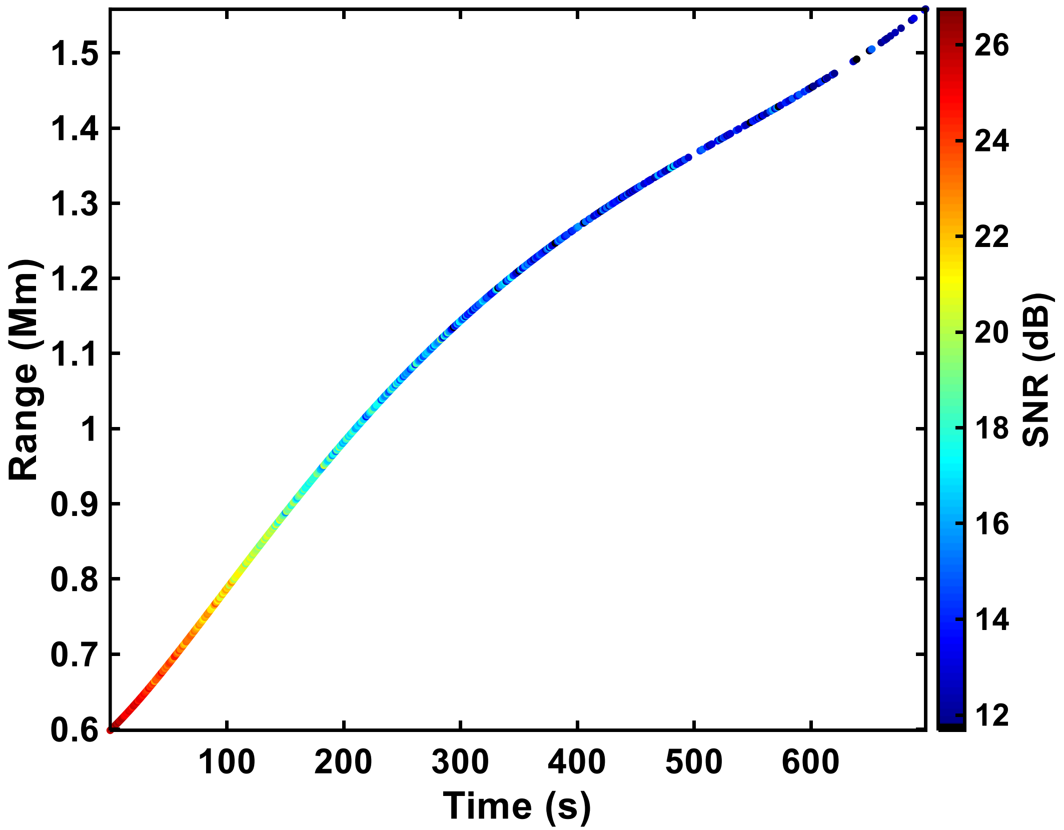


- **Example: Fusion node refiltering track states from a radar that is tracking a ballistic target with a maneuvering target filter**
  - State space is Earth-centered rotating position and velocity
  - Process noise model is spherically-distributed, white noise in the acceleration dimension
- **Fusion node receives local sensor track states, forms SSEMs with estimated process noise parameters, and then refilters**



Radar Measurement Model

Radar Detections



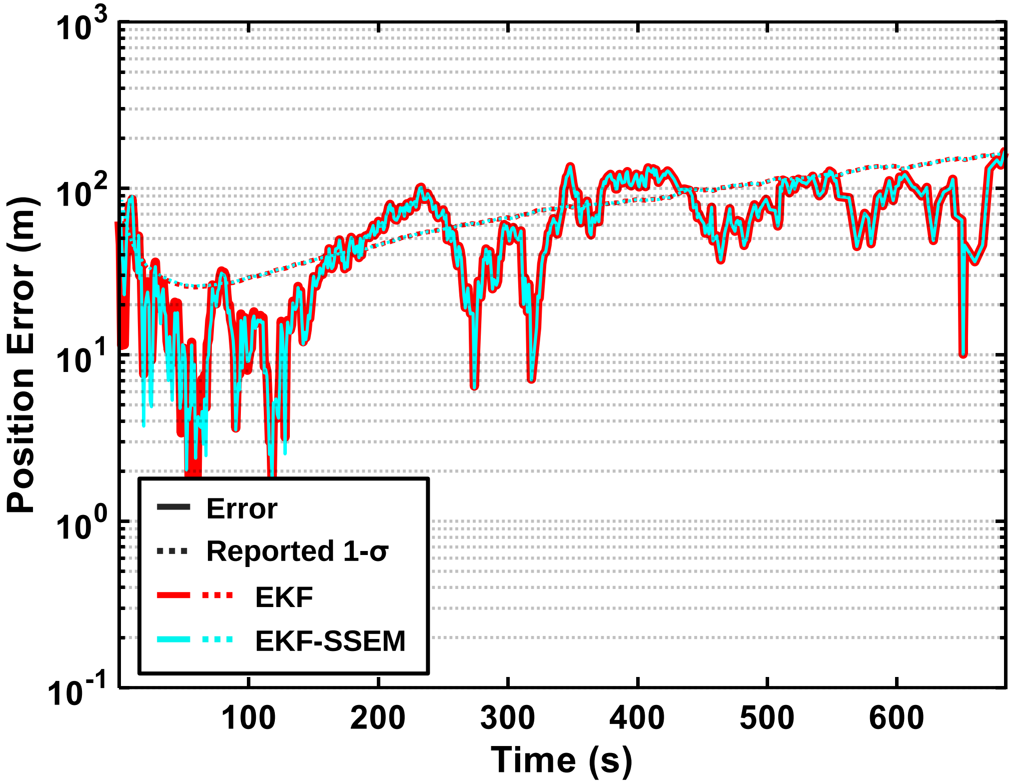
- **Target is assumed to have a radar cross-section (RCS) of 0 dBsm[1]**
- **Radar measurements are assumed to be from a phased-array-like system:**
  - Coordinate system is range-direction-cosine (RUV)
  - Range accuracy is dependent on radar bandwidth and signal-to-noise ratio (SNR)
  - Angle accuracy is dependent on radar beamwidth and SNR

Parameter	Value
Bandwidth	100 MHz
Beamwidth	1 mrad
Range: 0 dB SNR on 0 dBsm target	2700 km

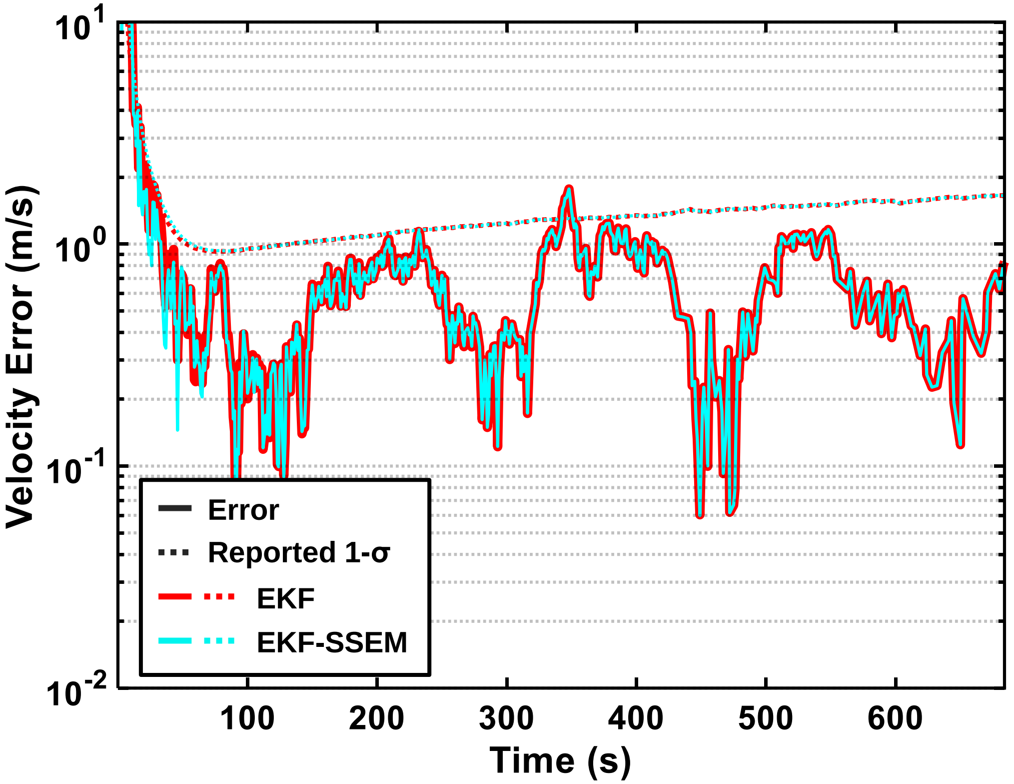
[1]Square meters in decibels

Refiltering Verification

Position Error Comparison



Velocity Error Comparison

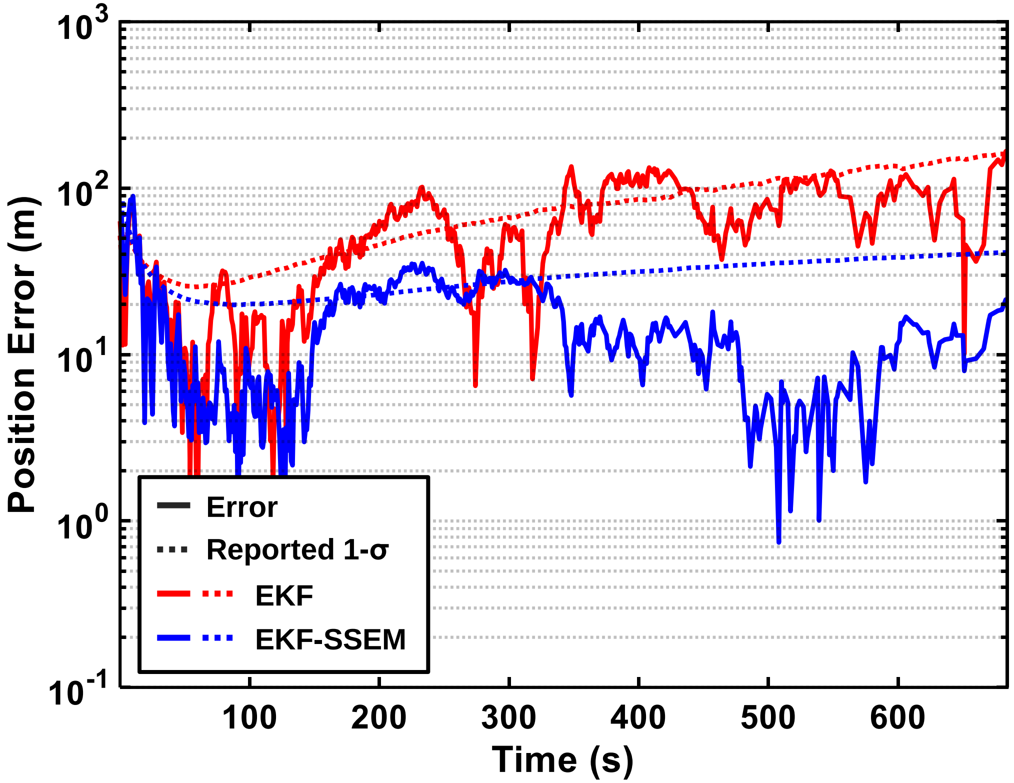


Reconstructed effective measurements with estimated process noise are sufficient to closely recreate original filtered result

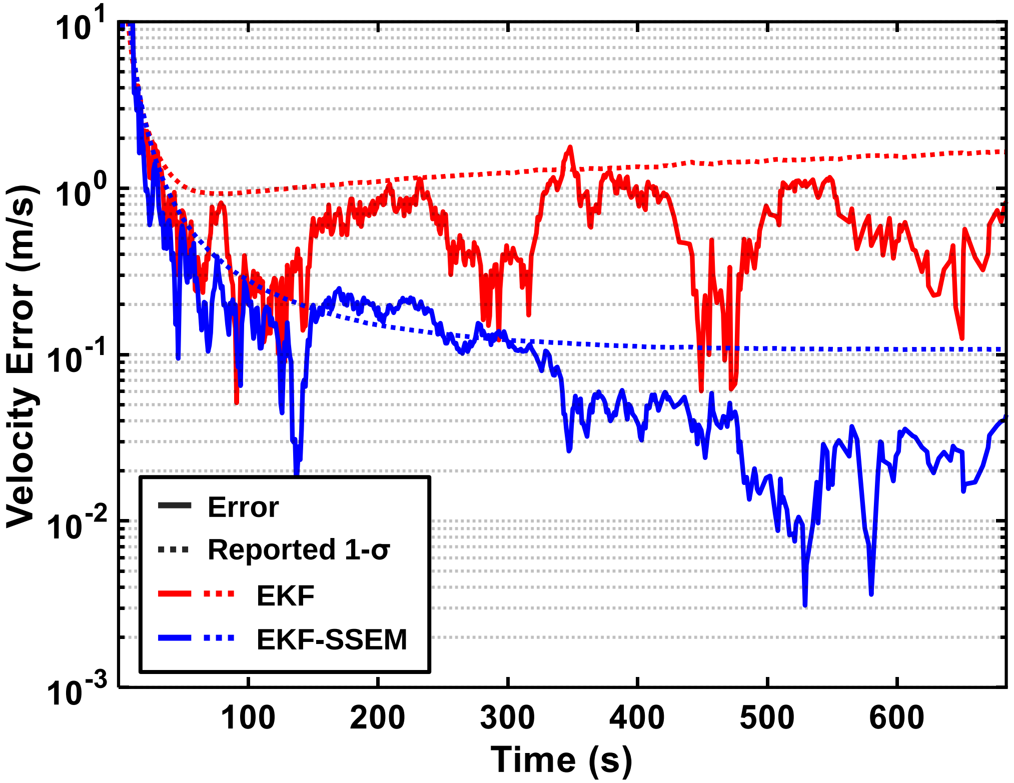
Local and refiltered process noise power spectral density: 0.01 m^2/s^3

Improvements from Refiltering

Position Error Comparison



Velocity Error Comparison



By refiltering with a dynamics-matched estimator, position and velocity estimates are appreciably improved

Process noise power spectral density: 0.01 m^2/s^3 (Local), 0.00001 m^2/s^3 (Refiltered)

Summary

- **Studied temporal decorrelation of state estimates at a fusion node to enable refiltering**
  - Allows for fusion node to exploit global information to tailor estimation process and/or fix local sensor tracking errors
- **Decorrelation process was developed for a fusion node with:**
  - Access to track states only
  - No knowledge of sensor measurement model
  - Known process noise model with unknown parameters
- **Effectiveness of decorrelation and refiltering shown through ballistic target tracking example**
- **Desired to extend to mismatched measurement and fusion rates (shown empirically)**

Fusion Node Rate 1/4 of Measurement Rate