Differentiable Point Scattering Models for Efficient Radar Target Characterization

Zachary Chance, Adam Kern, Arianna Burch, Justin Goodwin

MIT Lincoln Laboratory

MSS Tri-Service Radar Symposium

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited.

This material is based upon work supported by the Under Secretary of Defense for Research and Engineering under Air Force Contract No. FA8702-15-D-0001. Any opinions, findings, conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Under Secretary of Defense for Research and Engineering.

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

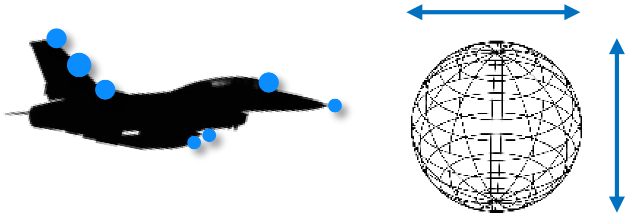
© 2021 Massachusetts Institute of Technology.

Delivered to the U.S. Government with Unlimited Rights, as defined in DFARS Part 252.227-7013 or 7014 (Feb 2014). Notwithstanding any copyright notice, U.S. Government rights in this work are defined by DFARS 252.227-7013 or DFARS 252.227-7014 as detailed above. Use of this work other than as specifically authorized by the U.S. Government may violate any copyrights that exist in this work.

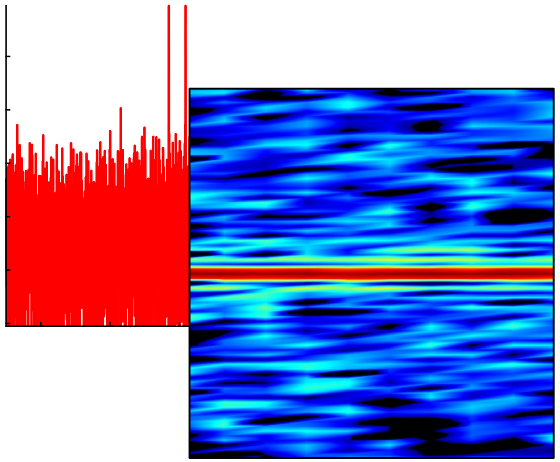
Radar Target Characterization

- Many defense applications require the extraction of target shape information from radar data for the purposes of characterization and classification
- Often approached using classical Fourier-based imaging techniques*
  - Pro: Efficient, well-understood
  - Con: Limited by traditional time-frequency relationships, e.g., bandwidth, aspect angle span
- Can also employ parametric target models that are fit to received data*
  - Pro: Offer higher resolution than classical methods
  - Con: Commonly require complex optimization
- In this work, differentiable point scattering models that offer high resolution *and* efficient optimization are studied

Target Models



Data Products



*Citations are included in accompanying paper

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Outline

- Introduction
- ➡ Differentiable Radar Scattering Models
- Examples
- Summary

Radar Signature Modeling

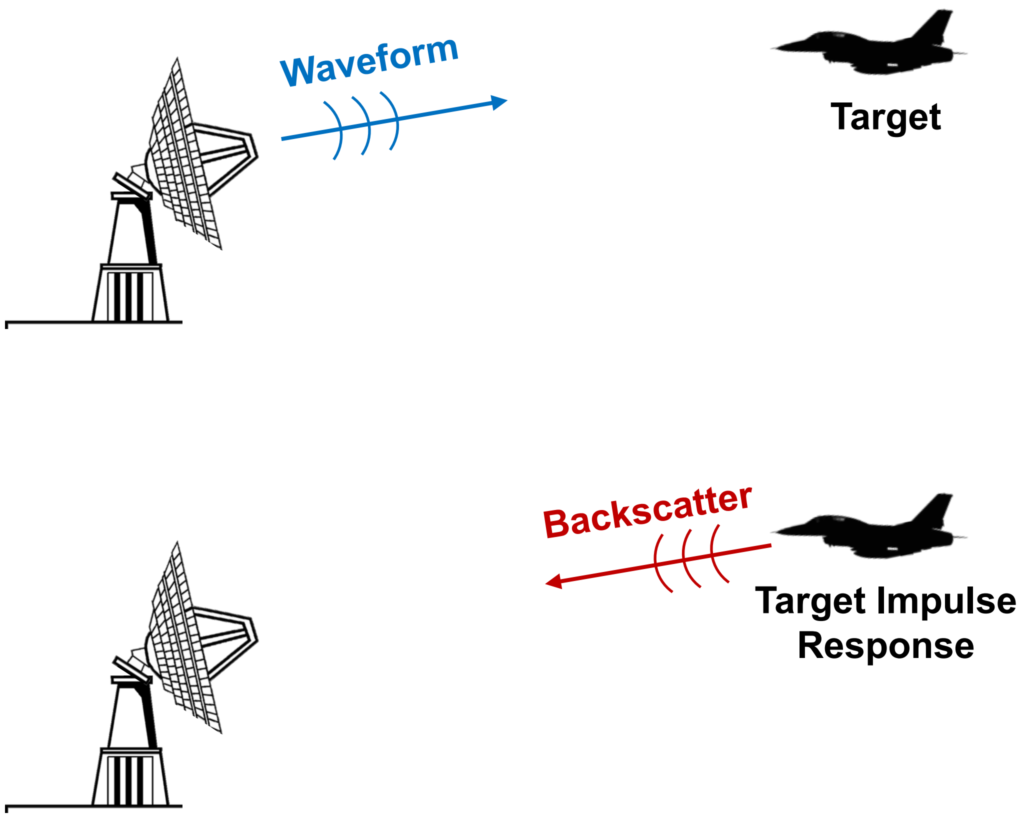
- For targets that slowly change viewing perspective[1], the received response from a radar transmission can be written as

$$y(t,\boldsymbol{\ell}) = x(t)e^{j2\pi f_c t} * h(t,\boldsymbol{\ell})$$

  - $x(t)$ is the transmit waveform
  - f_c is the transmit center frequency
  - $\boldsymbol{\ell}$ is the line-of-sight vector
  - $h(t,\boldsymbol{\ell})$ is the target impulse response

- A scattering model is a parametric target impulse response[2]

$$h(t,\boldsymbol{\ell};\boldsymbol{\theta})$$



[1]Relative to length of radar observation
[2]Frequency response also used

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Point Scattering Model

- A point scattering model is a composite scattering model of the form:

$$h(t, \boldsymbol{\ell}; \boldsymbol{\theta}) = \sum_{n=1}^{N} h_n(t, \boldsymbol{\ell}; \boldsymbol{\theta}_n)$$

- Each constituent impulse response is of the form[1]:

$$h_n(t, \boldsymbol{\ell}; \boldsymbol{\theta}_n) = a_n \delta(t + 2\mathbf{p}_n^T \boldsymbol{\ell}/c)$$

  - a_n is the amplitude of the point
  - $\mathbf{p}_n$ is the x-y-z location of the point

- Amplitude and position will be functions of parameter vectors $\boldsymbol{\theta}_n$ and line-of-sight $\boldsymbol{\ell}$

[1]Assumes target features have approximately constant amplitude frequency response over observed band

Differentiable Point Scattering Models

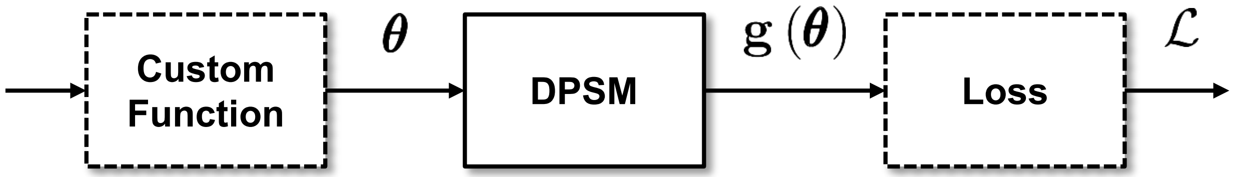
- A *differentiable point scattering model* (DPSM) is a point scattering model where each point's amplitude and position functions are differentiable with respect to the model parameter vector θ

$$h_n(t, \boldsymbol{\ell}; \boldsymbol{\theta}_n) = a_n \delta(t + 2\mathbf{p}_n^T \boldsymbol{\ell}/c)$$

Differentiable

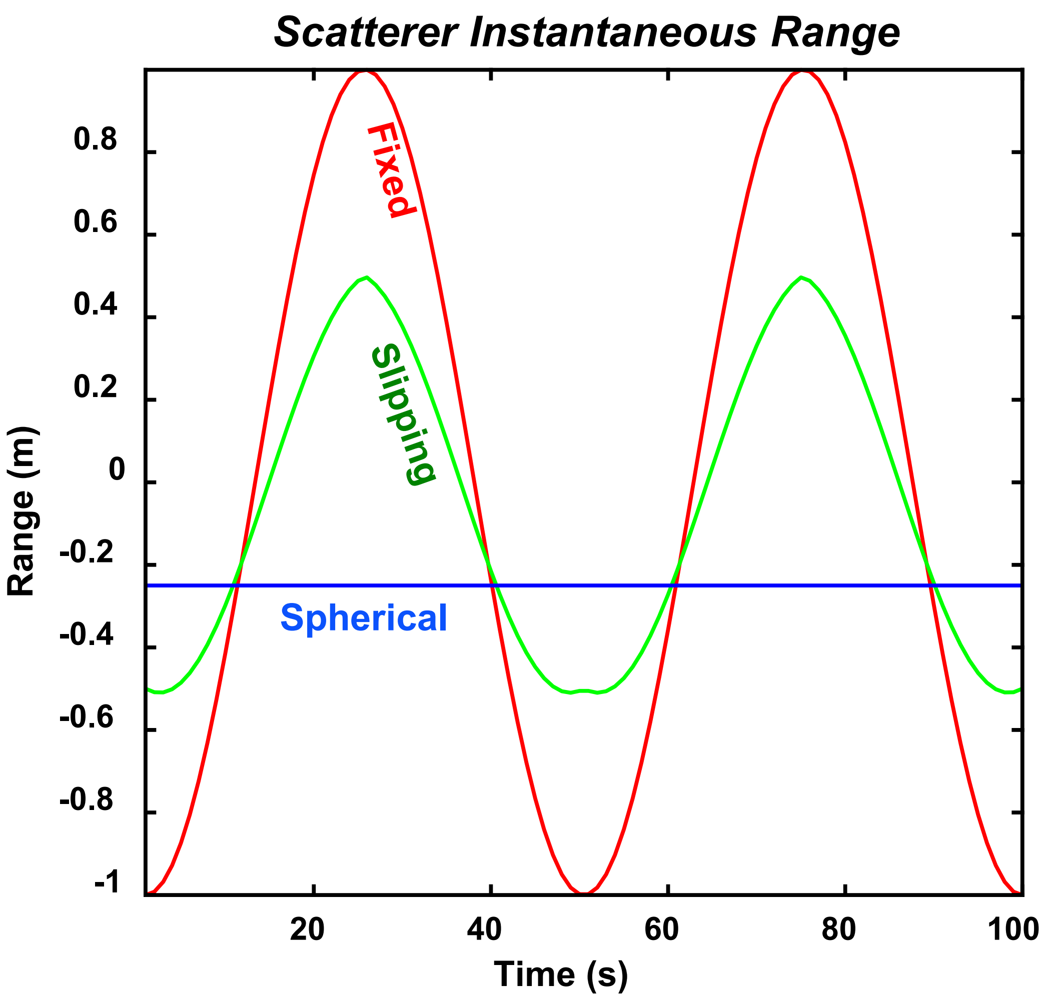
- A DPSM allows for the calculation of the gradient of data with respect to model parameters

- This can be chained with functions before (functions that create DPSM parameters) and after (functions of data, e.g., loss)



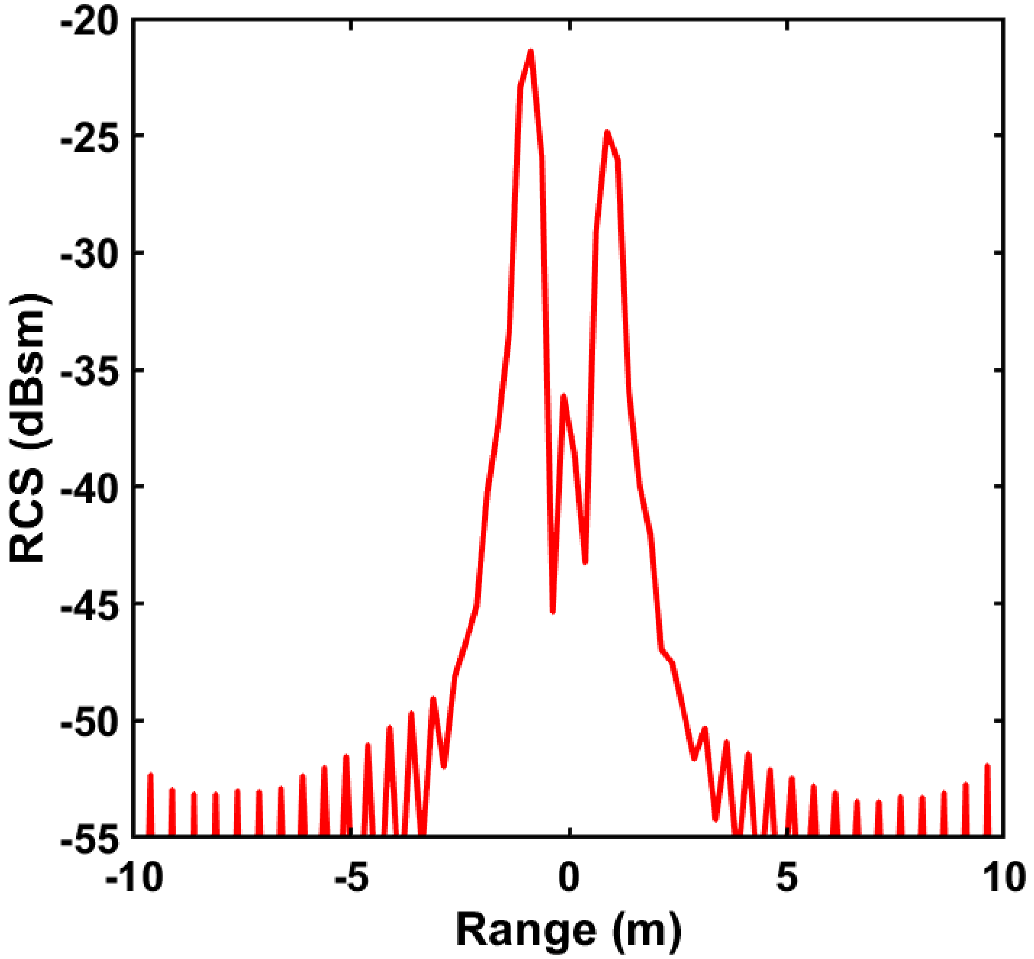
LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Example Differentiable Scatterer Types



- Some examples of scatterer common amplitude and position definitions that are differentiable:
  - Fixed amplitude, constant over all viewing perspectives
  - Fixed position, constant on target axis over all viewing angles
  - Slipping position, assumes closest point on ring centered around body axis
  - Spherical position, takes closest point on sphere located at target origin

Range Profile



- Common form of radar data used for demonstration is a range profile
- Can be shown to be of the form:

$$g(r, \boldsymbol{\ell}) = \sum_{n=1}^{N} \gamma_n a_n R_{xx}\left(2r/c + 2\mathbf{p}_n^T \boldsymbol{\ell}/c\right)$$

  - r is the range
  - γ_n is propagation phase delay
  - $R_{xx}(\tau)$ is the autocorrelation of the radar waveform $x(t)$
- Range profiles will be the main data sourced used for model optimization

LINCOLN LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Range Profile Gradient

- **Gradients of loss functions based on range profiles will depend on the gradient of the data with respect to the model parameter vector θ**
- **Using chain rule and independence of scatterer parameter vectors θ_n, the gradient of the range profile at a given range due to the n^th scatterer is**

$$\frac{\partial g(r,\boldsymbol{\ell})}{\partial \boldsymbol{\theta}_n} = \frac{\partial \gamma_n}{\partial \boldsymbol{\theta}_n} a_n R_{xx}(2r/c + 2\mathbf{p}_n^T \boldsymbol{\ell}/c) + \gamma_n \frac{\partial a_n}{\partial \boldsymbol{\theta}_n} R_{xx}(2r/c + 2\mathbf{p}_n^T \boldsymbol{\ell}/c) + \gamma_n a_n \frac{\partial R_{xx}(2r/c + 2\mathbf{p}_n^T \boldsymbol{\ell}/c)}{\partial \boldsymbol{\theta}_n}$$

First term: **Change in profile due to change in phase delay**

Second term: **Change in profile due to change in scatterer amplitude**

Third term: **Change in profile due to change in scatterer range**

- **These terms can all be calculated for DPSMs and radar waveforms that can be written analytically[1]**
- **DPSM codebase calculates these values for constructed models and given waveforms**

[1]**Waveforms can also be approximated using analytic functions**

Outline

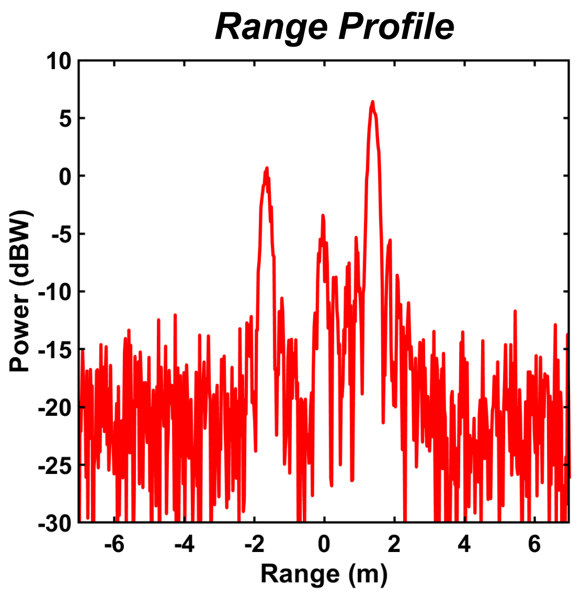
- Introduction
- Differentiable Radar Scattering Models
- ➡ Examples
- Summary

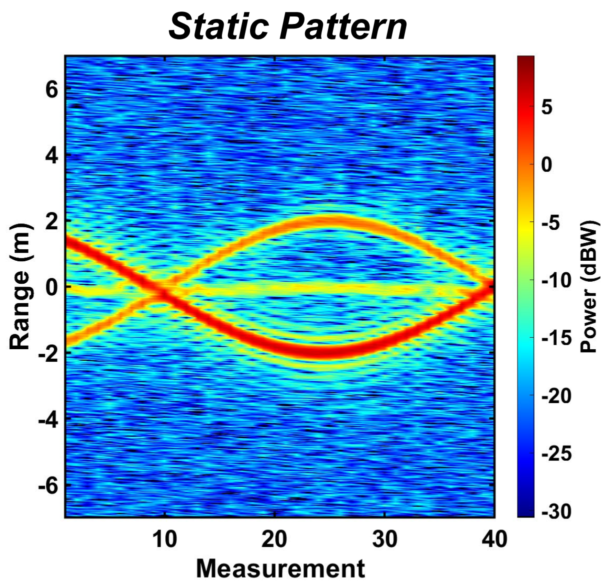
Simulated Examples

- **Example model consists of three scatterers:**
  - **Fixed scatterer**
    - **Amplitude: 1 + j0 (1.01 + j0)**
    - **Radial position: 0.5 m (0.6 m)**
    - **Azimuthal position: 0 rad (0 rad)**
    - **Axial position: 2 m (2.1 m)**
  - **Fixed scatterer**
    - **Amplitude: 2 + 0j (1.9 + j0)**
    - **Radial position: 0 m (-0.1 m)**
    - **Azimuthal position: 22.5° (22.5°)**
    - **Axial position: -2 m (-2.1 m)**
  - **Slipping scatterer**
    - **Amplitude: 0.5 + j0 (0.51 + j0)**
    - **Radial position: 0.1 m (0.1 m)**
    - **Axial position: 0 m (0.1 m)**

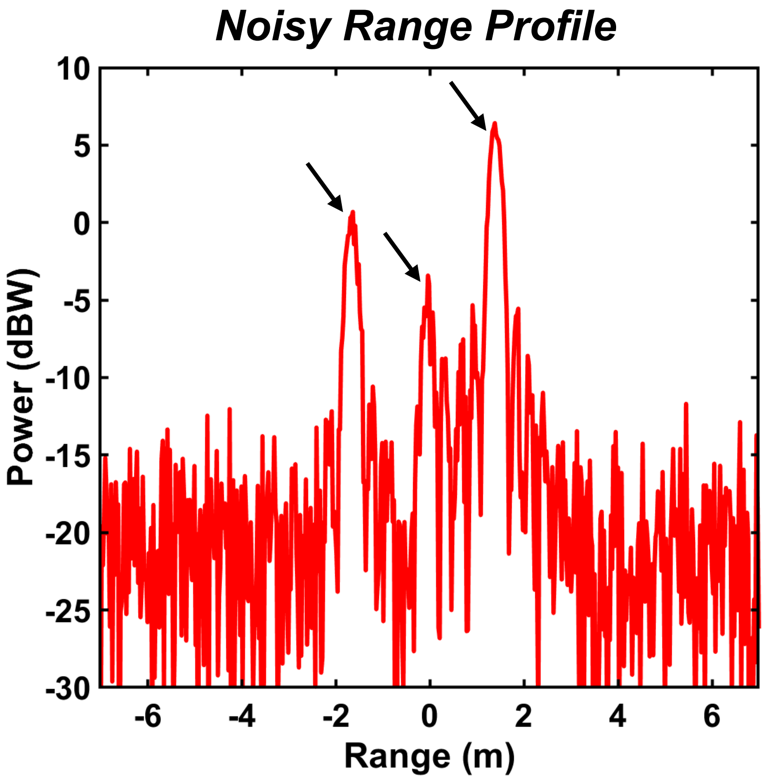
(Initial estimates)

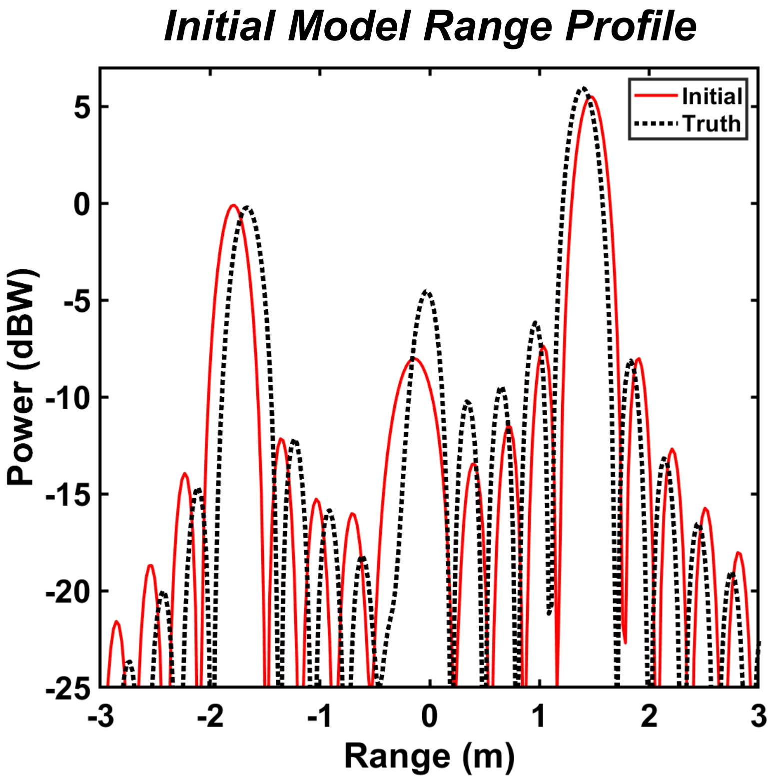
- **Radar parameters:**
  - **Bandwidth: 500 MHz**
  - **Waveform: Linear FM**
  - **Noise: -20 dBW**





Model Fitting for Range Profile

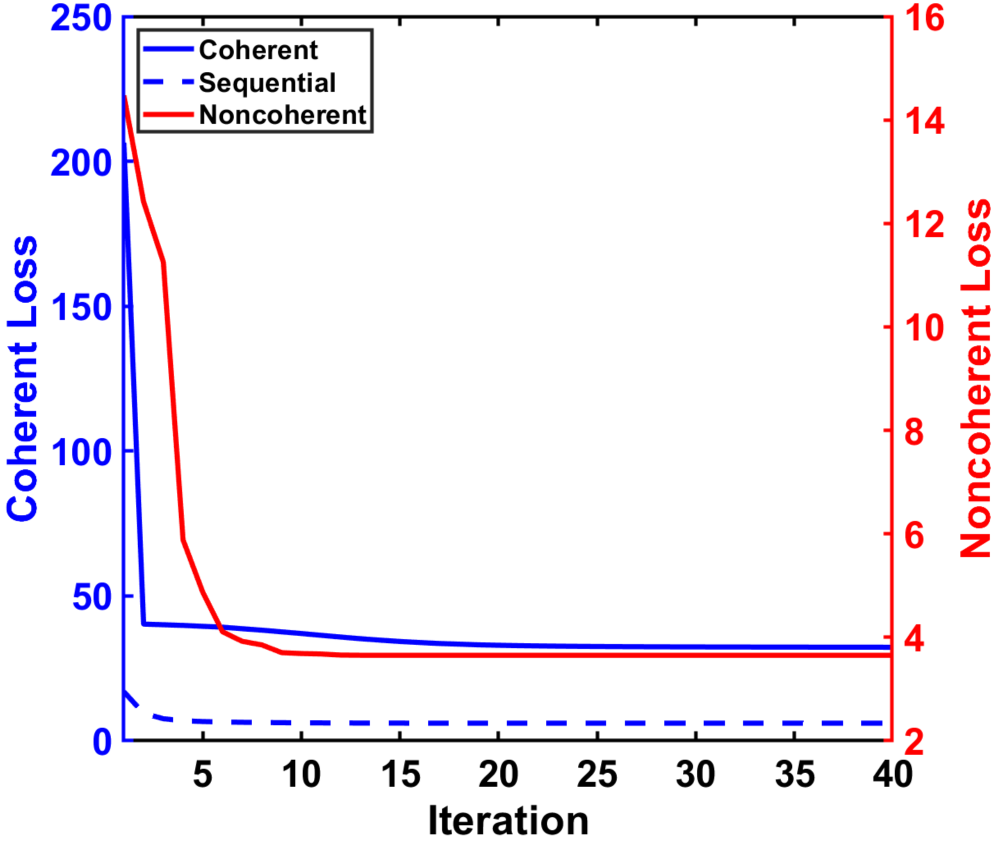




Measured range profile exhibits three distinct scatterers; initial model show appreciable differences from true model range profile

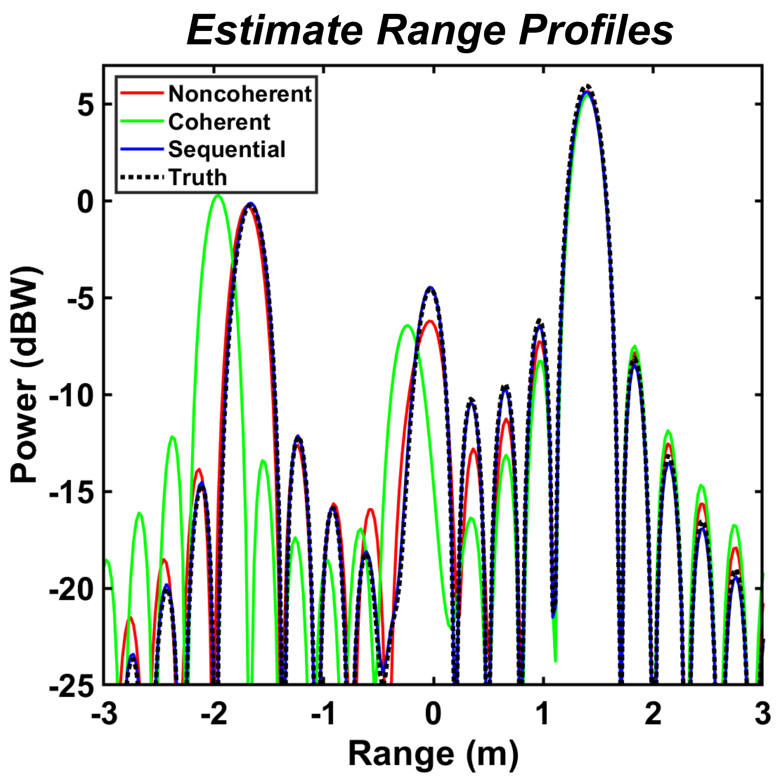
Model Fitting for Range Profile (cont'd)

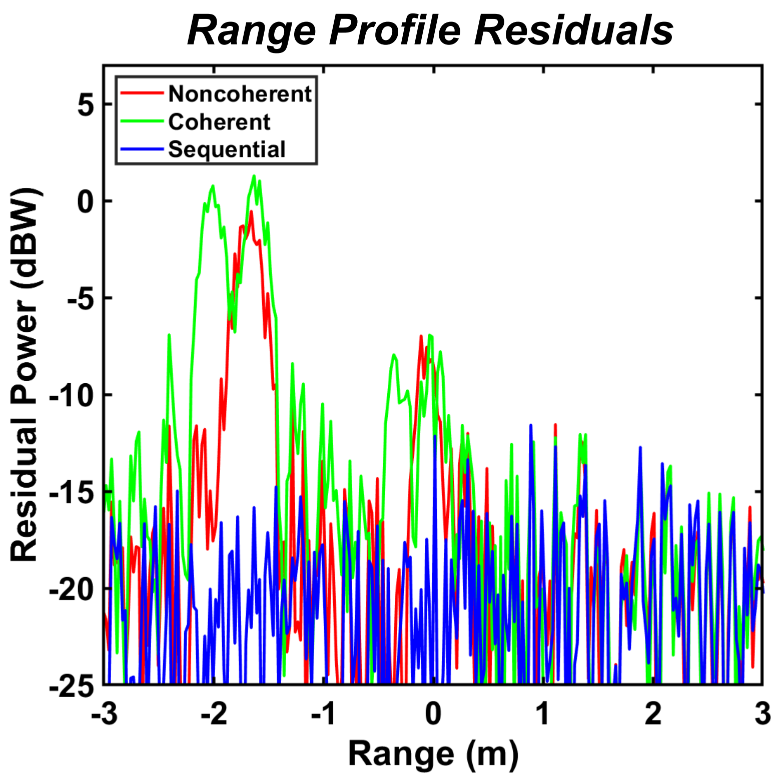
- Model parameters optimized using gradient descent
- Two common loss functions used
  - Coherent: Square error of hypothesized data relative to observed data
  - Noncoherent: Square error of amplitude of hypothesized data relative to amplitude of observed range profile
- Sequential optimization also considered (i.e., coherent optimization seeded by noncoherent)



Coherent optimization finds nonoptimal local minimum; sequential optimization successfully finds global minimum

Model Fitting for Range Profile (cont'd)

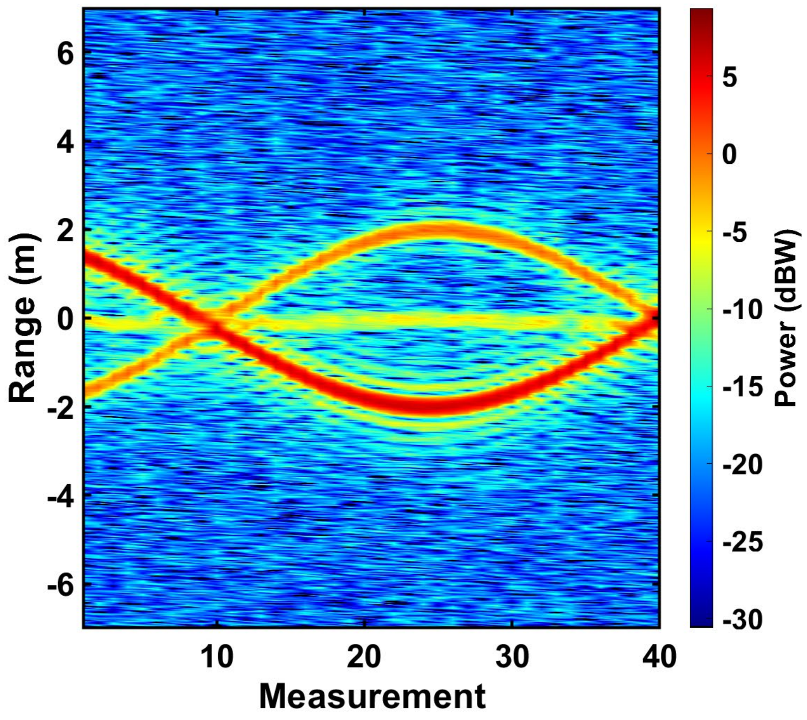




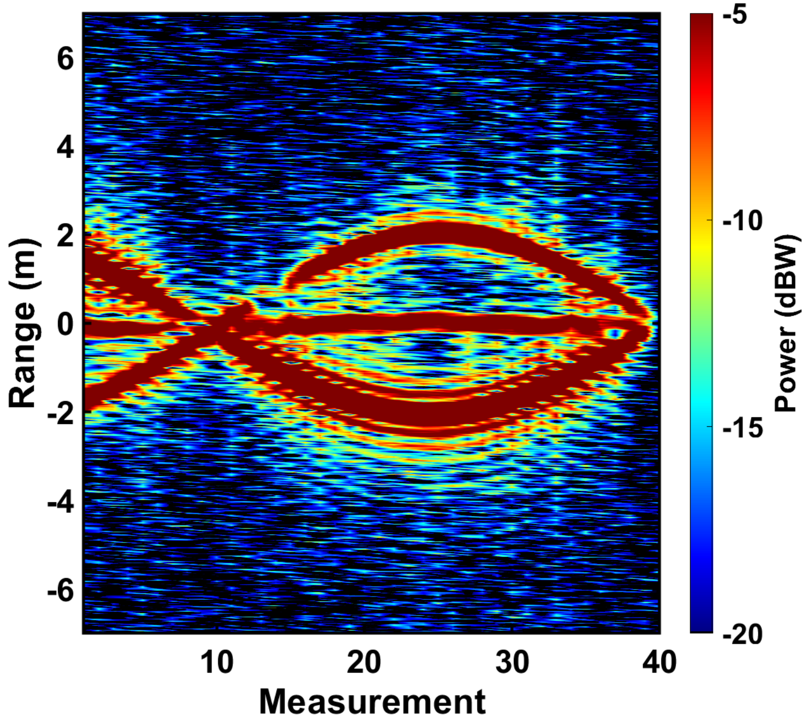
Sequential optimization allows for coherent estimation to effectively estimate target model parameters; residual power is on the order of the additive noise power

Model Fitting for Static Pattern

Noisy Static Pattern

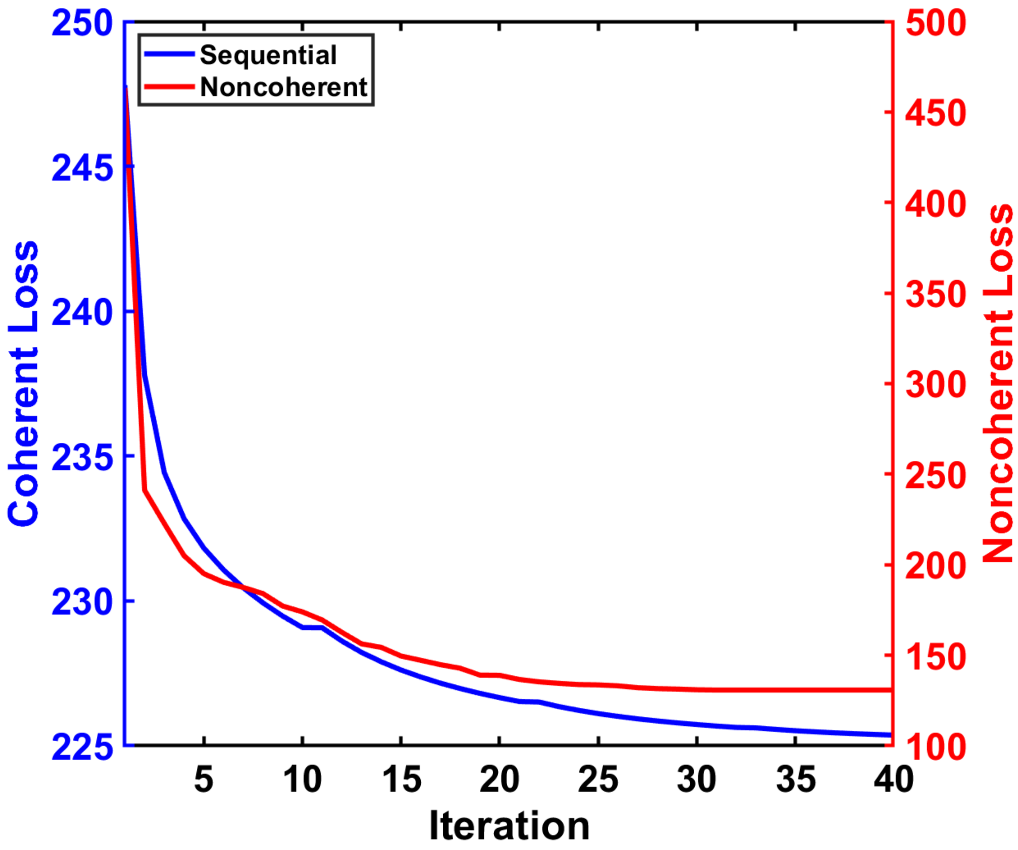


Initial Model Residual Image



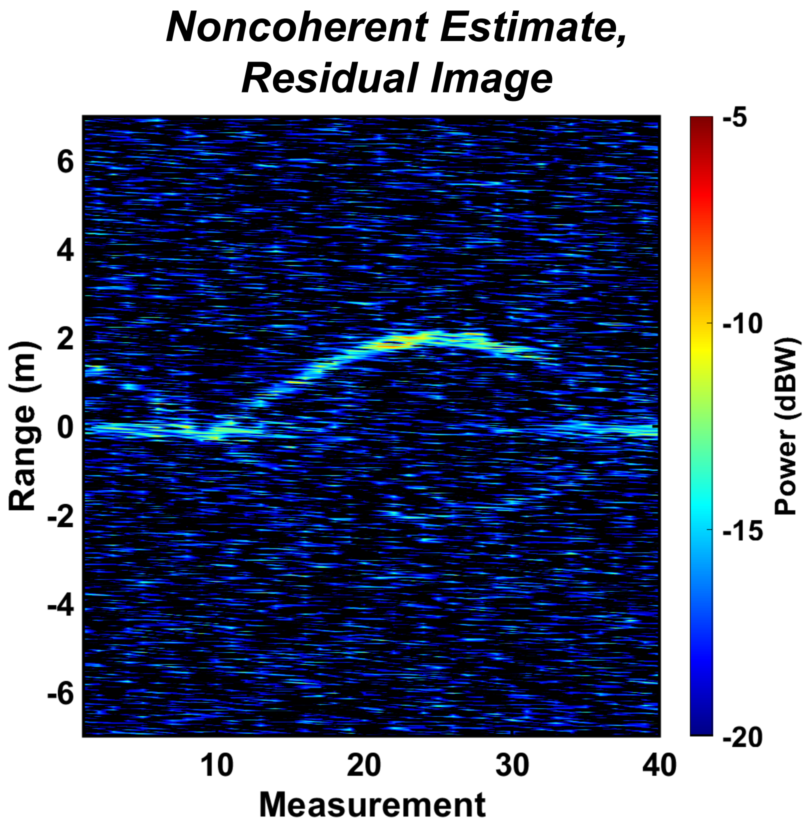
Initial model shows significant differences over all observed viewing angles

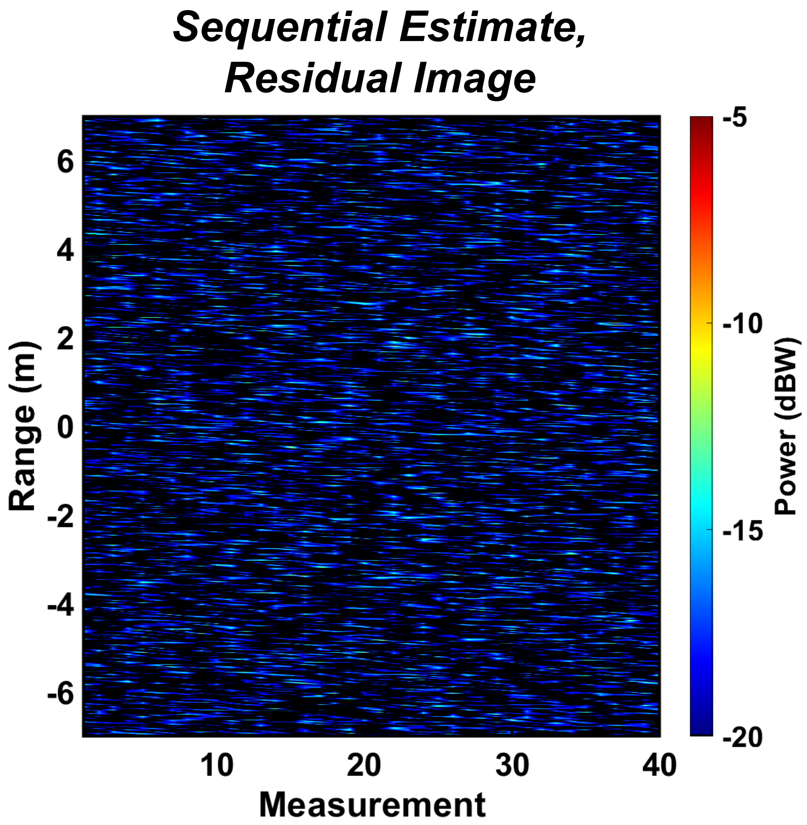
Model Fitting for Static Pattern (cont’d)



Gradient descent allows for intelligent refinement of target model parameters for various loss functions

Model Fitting for Static Pattern (cont'd)





Noncoherent estimation coarsely honed target model estimate; sequential estimation was able to accurately estimate scattering model parameters to leave a residual on the order of the additive noise

Summary

- Radar target characterization is an important component of many defense missions
- Fitting radar signature models to data can enable high resolution target estimates that can be utilized for characterization
- Efficient fitting and performance prediction can be afforded by differentiable point scattering models by exploiting gradient information
- Future work will investigate construction of initial estimates for seeding gradient descent algorithms along with incorporation of non-point-like scatterers

UNCLASSIFIED